Comparative Welfare in Deferred Acceptance

Consider a market which consists of M men and W women. Each man m_i where $i = 1, ..., M$ has value $m_i^j \sim Uni(0,1)$ for woman w_j, and each woman w_j has value $w_j^i \sim Uni(0,1)$ for man m_i. m_i^j is independently and identically distributed from m_i^ℓ where $1 < \ell \neq j < W$; likewise, it is independently and identically distributed from m_k^j where $1 < k \neq i < M$. Values are independently and identically distributed for women in the same way. There is demand only for a single partner and preferences are strict. No utility is derived from an empty match.

The assignment mechanism analyzed here is men-proposing deferred acceptance: (0) Each person ranks those of the opposite gender in accordance with their preferences for a marriage partner. (1) Each man proposes to the woman for whom he has the highest value. (2) If a woman receives more than one proposal, she "holds" the most preferred and rejects all others. (3) Any man rejected at step $k-1$ makes a new proposal to his most preferred woman who hasn't yet rejected him. (4) Each woman holds her most preferred acceptable offer to date, and rejects the rest. The algorithm stops when no further proposals are made, at which point a match is created between each woman and the man whose proposal she is holding.

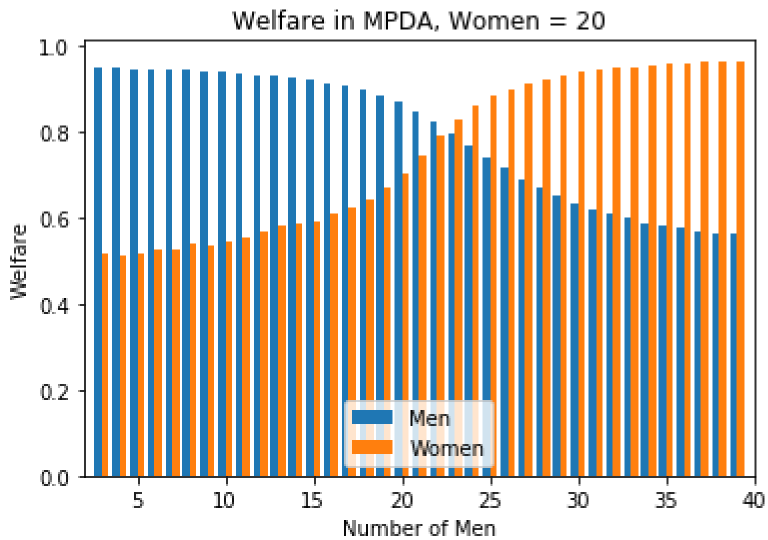
This note compares average welfare statistics for different market sizes. First, I assess average utility for both men and women when $W = 20$ and $M = [3, 40]$. Then, I vary $W = [3, 40]$ as well and draw conclusions on the ways in which individuals can maximize personal welfare (e.g., by decreasing competition or by proposing instead of being proposed to).

With 1,000 iterations, the average utility of each man and each woman in given market conditions is shown in **Figure 1**. Note that only the utility of each market member who is matched is calculated in the average; every unmatched member receives utility 0 from the market.

The average utility of each man strictly decreases in the number of men, and the average utility of each woman strictly increases in the number of men. This is intuitive: increased competition decreases one's chances at arriving at a good choice. Trivially, the average utility of any matched market member never falls below 0.5; in worst case scenario a market member receives $\mathbf{E}(Uni(0,1)) = 0.5$.

It is clear as well that the difference in utility between the proposing and non-proposing side is significant, except when the number of men is slightly greater than the number of women, in this case at $M = 23$ and $W = 20$. Here, the increase in competition contributes to enough utility loss for men that it

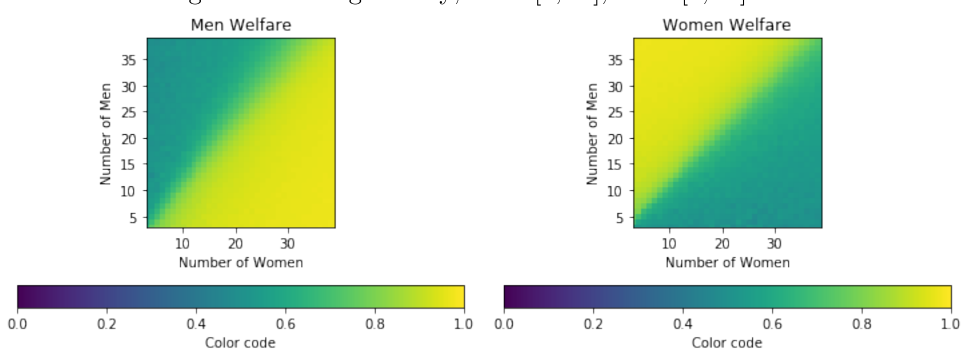
Figure 1: Average utility; $W = 20$, $M = [3, 40]$



neutralizes the utility gain from proposing.

In **Figure 2**, I analyze average utility of each market member when the number of men varies.

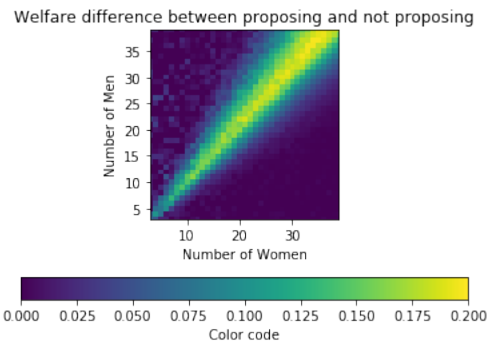
Figure 2: Average utility; $W = [3, 40]$, $M = [3, 40]$



It still holds that the utility of each man decreases in the number of men; it is true by the same logic that it increases in the number of women. Likewise women utility decreases in the number of women and increases in the number of men. The difference in proposing is clear: we saw above that $M = 20$ and $W = 20$ generated average utility of men slightly above that of women; the same is evidenced here.

I found it useful to evaluate the difference in utility between proposing and non-proposing sides at every possible market size wherein $M = [3, 40]$ and $W = [3, 40]$. See **Figure 3**.

Figure 3: Welfare difference between proposing and non-proposing sides; $W = [3, 40]$, $M = [3, 40]$



The difference in welfare of proposing and non-proposing market members is small when the size of one side of the market is significantly larger than the size of another. Observe the same trend in **Figure 2**: if the size of one side of the market is significantly larger than the other, the utility of a member of the larger side is roughly independent of its ability to propose. This is sensible: If proposing, a member of the smaller side picks the best of her possible matches, and faces little competition for that member; it is likely as well that her value for her follow-up choices is still larger than 0.5. If not proposing, a member of the smaller side can choose from a lot of possible matches, and is likely to arrive at her optimal choice.

However, the difference in welfare between the proposing and non-proposing side is large when the sizes of each side of the market is comparable. Observe this in **Figure 1** as well: around $M = 20$ and $W = 20$, the difference in average utility is around 0.15.

In **Figure 4** I analyze the difference in average utility when a member of one's market is removed, slightly decreasing competition for one's preferred match.

Note the change in the color code: the differences in utility from removing a member of one's market is often more insignificant (especially along $M = W$) than transitioning into a proposing side of the market, or vic versa. Removing a market participant from one's side hardly impacts outcomes when the number of participants is large.

There are two clear ways to improve individual welfare: to propose instead of being proposed to, and to decrease competition. Based on the previously used market conditions, I graph in **Figure 5** below which of the two methods is more optimal.

As is in line with previously discussed results, the rights to propose impacts individual utilities more when the sizes of the markets are comparable. From

Figure 4: Welfare difference when removing a member of one's market; $W = [3, 40]$, $M = [3, 40]$

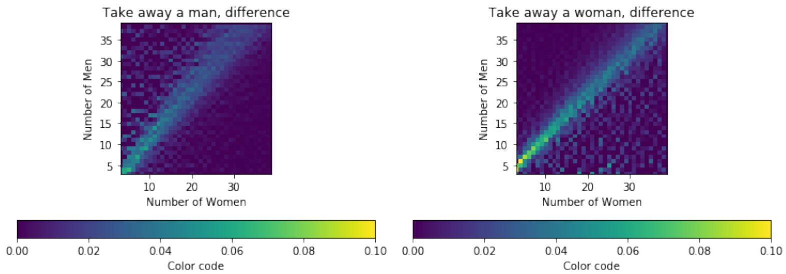


Figure 5: Optimizing outcomes: proposing versus removing competition; $W = [3, 40]$, $M = [3, 40]$

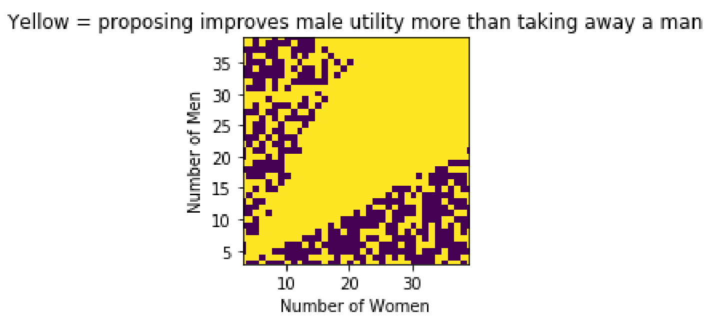


Figure 3 and **Figure 4**, it appears it is difficult for the short side of the market to drastically improve upon its individual utilities; **Figure 5** however shows that decreasing competition is sometimes more optimal than swapping rights to propose when the markets have largely different sizes.

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Our next concern is for the proportion of men for whom the utility gain from MPDA is stronger (and weaker) than that gained from a fixed reference man leaving the market.

I first present the impact of one-less [constant reference] man on the welfare of men in WPDA. All following data are aggregated over 50 iterations. In **Figure 6** the fraction of men who benefit from one-less man in the market are shown in red; who fare worse in blue; and who fare the same in green.

The benefits of decreased competition are strongest when the number of men is roughly similar to the number of men in the market, nonetheless it is beneficial in aggregate at every market size.

In **Figure 7** I analyze the same market but instead of taking a way a reference man, I grant proposing rights to men (as in MPDA).

The trend in **Figure 7** has previously been explored: the benefits of propos-

Figure 6: WPDA: $W = 5, 10, 15, 20$; $M = [4, 40]$

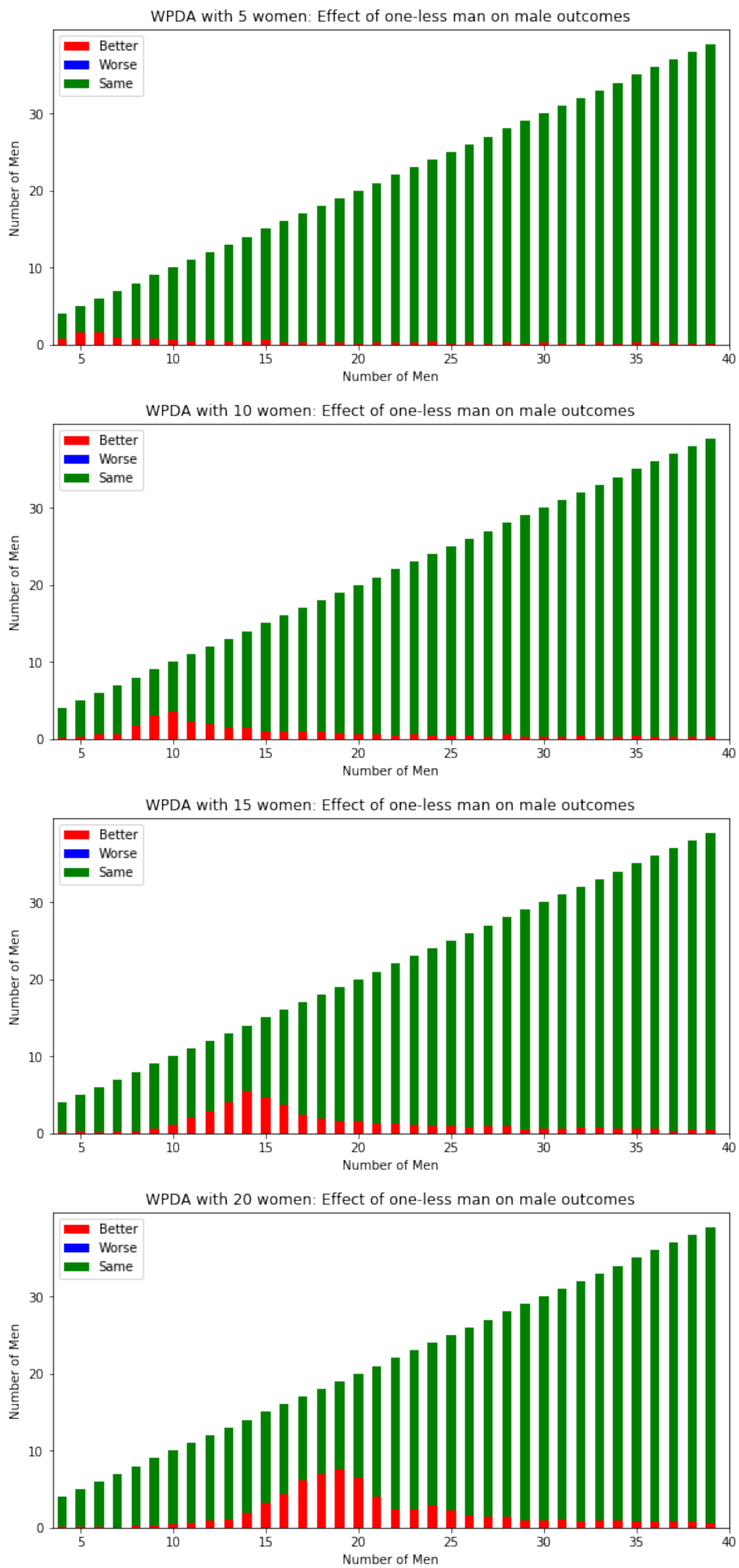
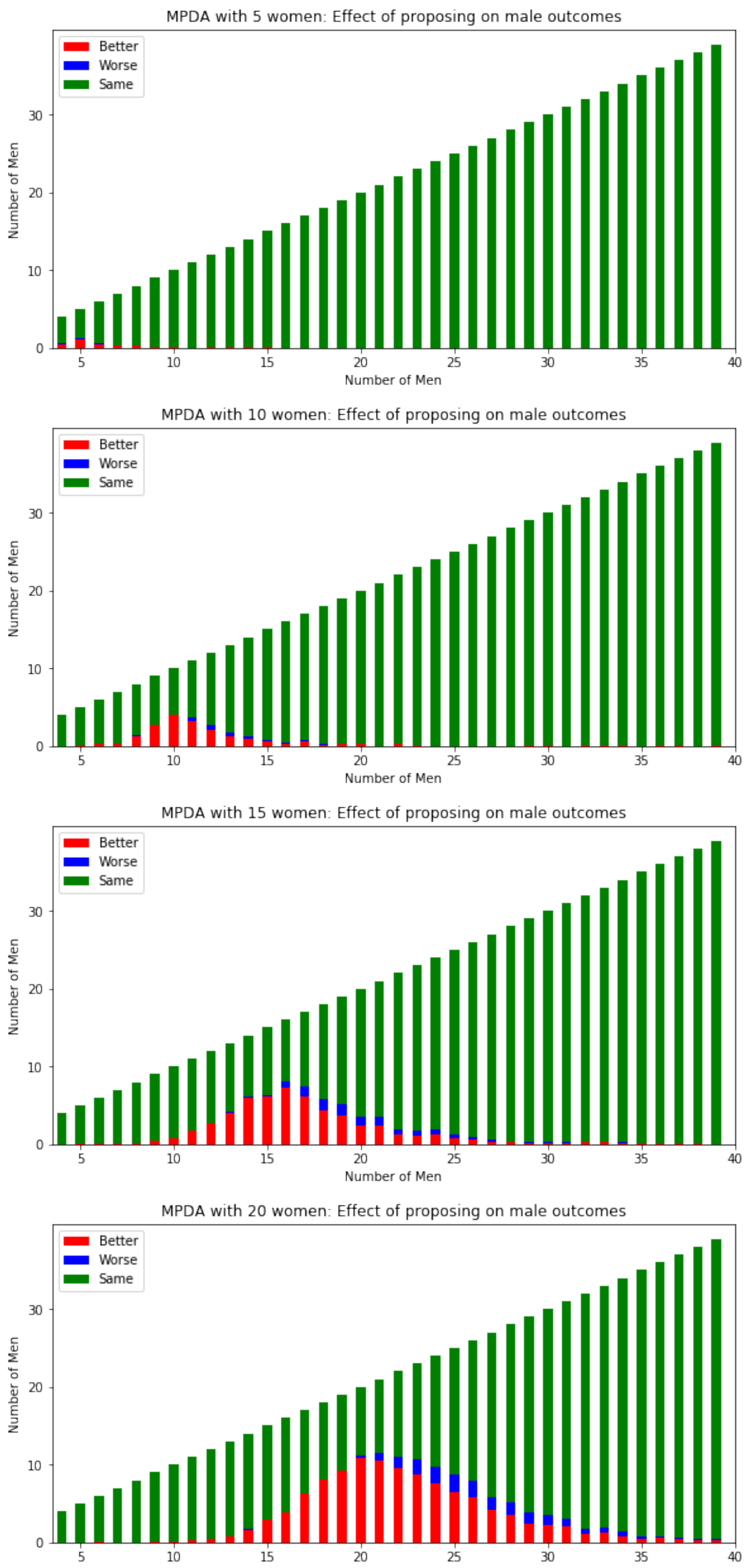
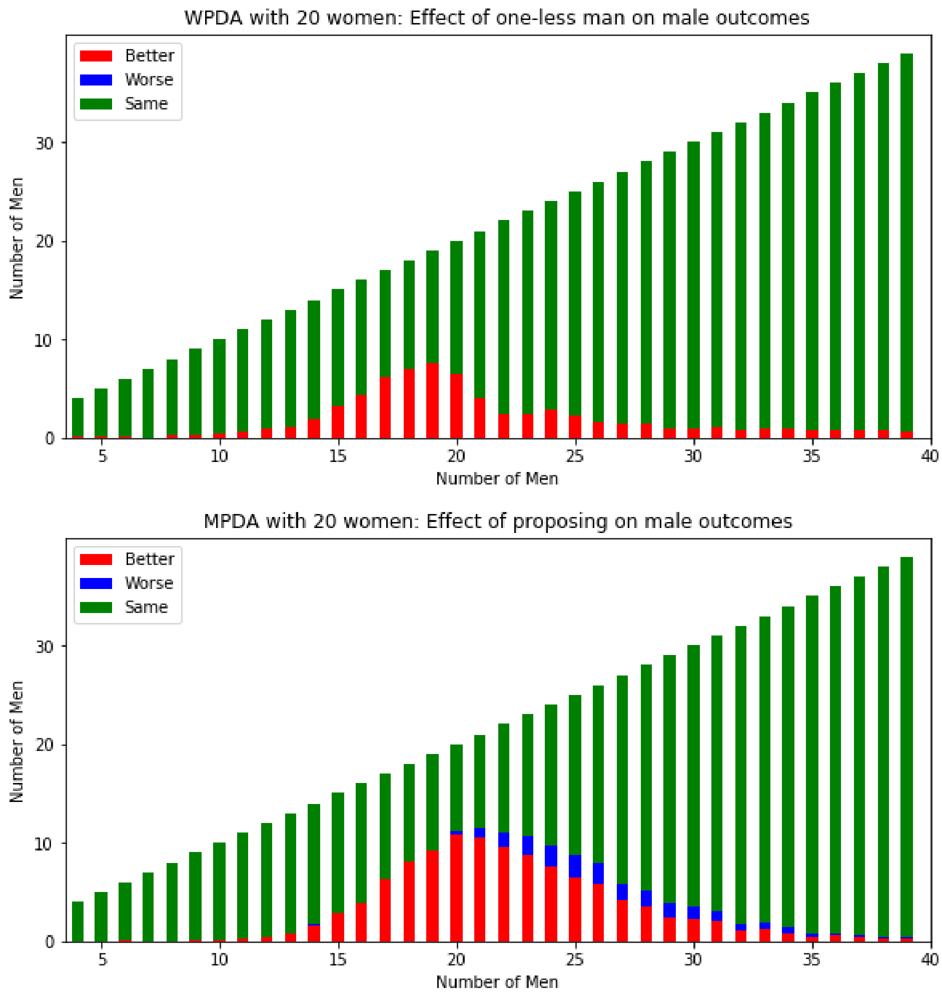


Figure 7: MPDA: $W = 5, 10, 15, 20$; $M = [4, 40]$



ing are strongest when market sizes are roughly equal. It now may be beneficial to compare these welfare effects side by side, for example when $W = 20$ and $M = [4, 40]$. See **Figure 8**.

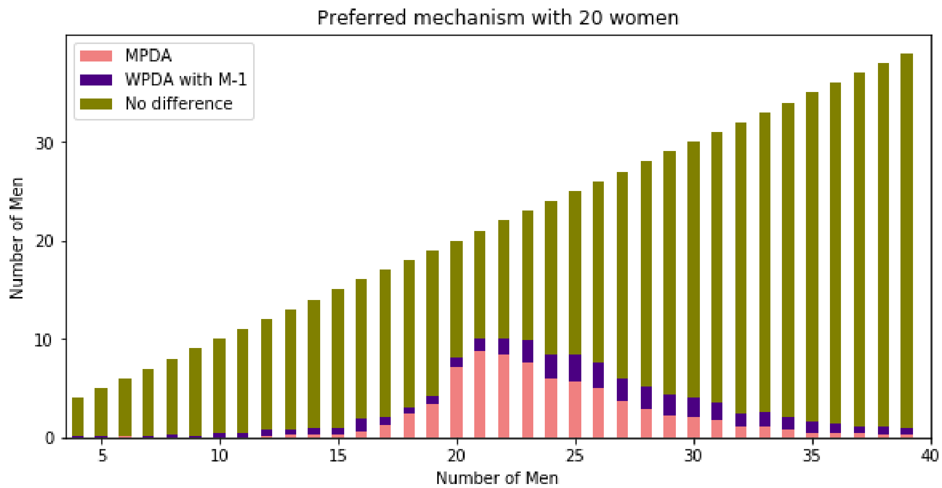
Figure 8: Comparing mechanisms; $W = 20$; $M = [4, 40]$



We see similar trends but a greater number of men seem to benefit from proposing rights, meanwhile some men are disadvantaged (in casualty) of proposing rights being granted to other men. In **Figure 9**, I now show the proportion of men wherein the utility of each in MPDA minus the utility in WPDA is smaller, equal to, and larger than the utility gain from having a fixed other reference man leave the market. First observe the case with $W = 20$, as in **Figure 9**.

MPDA is strongest when market sizes are similar, while decreased competi-

Figure 9: Fractions of men who prefer MPDA to $M-1$; $W=20$; $M=[4,40]$



tion may be preferred nearest the bounds. This shows the same trend as that of **Figure 5** but demonstrated with relative effects.

Consider the same but with $W=5,10,15$, as in **Figure 10**.

This demonstrates the proportion of men for whom the utility gain from MPDA is stronger (and weaker) than that gained from a fixed reference man leaving the market.

Figure 10: Fractions of men who prefer MPDA to $M-1$; $W=20$; $M=[4,40]$